About the Nature of the Recirculation Zone behind the Mach Disc in an Underexpanded Jet

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Abstract. The paper is devoted to the nature of the recirculation zone behind the Mach disc in an underexpanded jet predicted by CFD at some range of determining parameters. The free jet flow behind the axisymmetric sonic nozzle was simulated by two approaches – in the frames of full set of Navier-Stokes equations and by DSMC-approach. No recirculation zone predicted by NS-algorithm was observed in the flow field obtained by DSMC-approach. The nature of the discussed effect, therefore, seems to be purely computational and caused by limited applicability of the Navier-Stokes equations for the description of the flow in the vicinity of the Mach disc with high gradients of the flow parameters here.

Keywords: Underexpanded free jet, Mach disc, recirculation zone, Navier-Stokes equations, DSMC.
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INTRODUCTION

In some papers devoted to CFD study of the flow in an underexpanded free jet, the effect of the formation of the recirculation zone behind the Mach disc was discovered [1-6]. This zone represents a toroidal vortex in the near axis region with negative axial velocity here. Till now the effect seems not to have experimental confirmation. As it is noticed in [5] "…at the present state of research it cannot be excluded that the vortex might be of computational origin as a consequence of the (*r*, *z*) problem formulation, where screwlike three-dimensional flows are not possible". There is another possibility for computational nature of the discussed effect – limited applicability of the Navier-Stokes equations for the description of the flow in the vicinity of the Mach disc with high gradients of the flow parameters here.

To clarify the situation, the axisymmetric flow in an underexpanded jet behind the sonic nozzle was simulated by two approaches, namely, in the frames of the full set of unsteady Navier-Stokes equations, that were solved numerically by an original algorithm based on a staggered grid (NS – algorithm) [7], and in the frames of the standard DSMC – algorithm [8].

PROBLEM FORMULATION

The axisymmetric domain of simulation the flow field in underexpanded free jet is schematically shown in Fig. 1. It includes permeable boundaries 1, 3, 4, solid boundary 2 and the flow centerline 5. Through surface 1 representing the nozzle exit the gas inflow inside the domain takes place. The boundary conditions on surfaces 1-5 were set in accordance with the approach applied for simulation the flow field.

NS-Algorithm

The NS-algorithm representing the numerical method for solution of unsteady Navier-Stokes equations for compressible perfect gas based on the staggered grid is now well tested and approved. It allows one to simulate a wide variety of problems of sub-, trans- and supersonic gas dynamics [6, 7, 9].

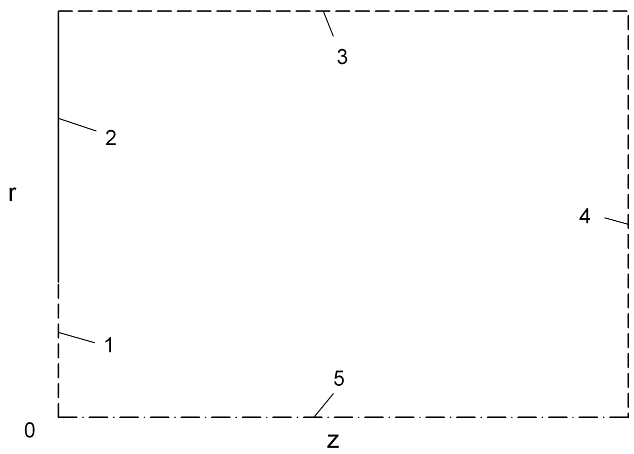

FIGURE 1. The domain of simulation the flow field in underexpanded free jet.

For NS – algorithm the set of determining parameters was as follows: the specific heats ratio $\kappa = 1.4$, the pressure ratio $p_0 / p_\infty = 400$, the temperature ratio $T_0 / T_\infty = 1$, the Reynolds number $\mathrm{Re} = \rho_0 c_0 r_e / \mu_0 = 1500$, the Prandtl number Pr = 0.71, the temperature dependence of viscosity $\mu \sim \sqrt{T}$, the bulk viscosity $\mu' = 0$. The domain of simulation shown in Fig. 1 was rectangular in shape with dimensions of $30\,r_e$ and $60\,r_e$ in radial and axial directions, respectively. The uniform grid with mesh sizes $dr = dz = 0.1 r_e$ was used. Here indexes $_0$, $_\infty$ and $_e$ are used to denote the variables in stagnation chamber, in the background gas and on the nozzle exit, respectively, c is the sound speed.

The considered free jet was assumed to be sonic with uniform distribution of parameters on surface 1. The temperature of solid surface 2 was assumed to be the same as stagnation temperature ($T_w = T_0$). At this surface the normal component of total velocity (w) was prescribed to be equal to zero while the tangential component (u) as well as the temperature T were defined by taking into account the velocity slip and temperature jump [10] with unit accommodation coefficients. In the developed NS-algorithm there is no need to prescribe the pressure or the density at the solid boundaries.

The conditions at the outlet boundaries (3, 4) were defined in the same manner. The axial and radial velocities were obtained by extrapolation from internal points of the domain. The same extrapolation was applied to the density and temperature for the outlet fluxes on these boundaries. For the inlet fluxes the density and temperature were defined assuming the total enthalpy and total pressure to be the same as in the flooded space. At the flow centerline the radial velocity as well as the radial derivatives of the axial velocity and temperature were prescribed to be zero.

DSMC-Approach

The simulation of the flow with the same set of determining parameters by DSMC-approach [8] was performed on the same grid used in NS-algorithm with about $2.5 \cdot 10^6$ simulated molecules in the domain of simulation at the steady stage. The simulation was performed without introducing the radial weighting factors. All the collisions between the molecules were assumed to be inelastic.

As it is known, the boundary conditions for DSMC-approach consist of prescribing the distribution function for only the molecules incoming in the domain [8]. Therefore, for surface 1 (see Fig. 1) this procedure is straightforward. On solid surface 2 the full accommodation of translational and internal degrees of freedom of incident molecules to the equilibrium values corresponding to the surface temperature was assumed. The setting of boundary conditions on permeable boundaries in DSMC-approach is not a trivial problem. On surface 3 the equilibrium distribution function typical of background gas was assumed – this assumption seems to be quite appropriate for the considered flow since the expected velocities of the gas on this surface are small compared to the sound speed. The same condition applied to surface 4 results in the appearance of strong disturbances of the flow parameters in some vicinity of this surface due to sufficiently high velocities of the gas here. Two ways to overcome this problem were tested.

One of them (BC1) consists of enlarging the axial size of the domain to the length providing the independence of the flow parameters at the desirable region of the flow from the conditions on surface 4. Test calculation shows that extending the axial size of the domain to the value of $80\,r_e$ is quite sufficient – the axial size or the region of mentioned disturbances was found to be $\leq 20\,r_e$.

Another way (BC2) consists of utilizing at surface 4 placed at the distance $z = 60\,r_e$ the radial profiles of parameters obtained by NS-algorithm. Both of these ways predicted similar distribution of flow field parameters inside the region $z \leq 60\,r_e$.

RESULTS AND DISCUSSION

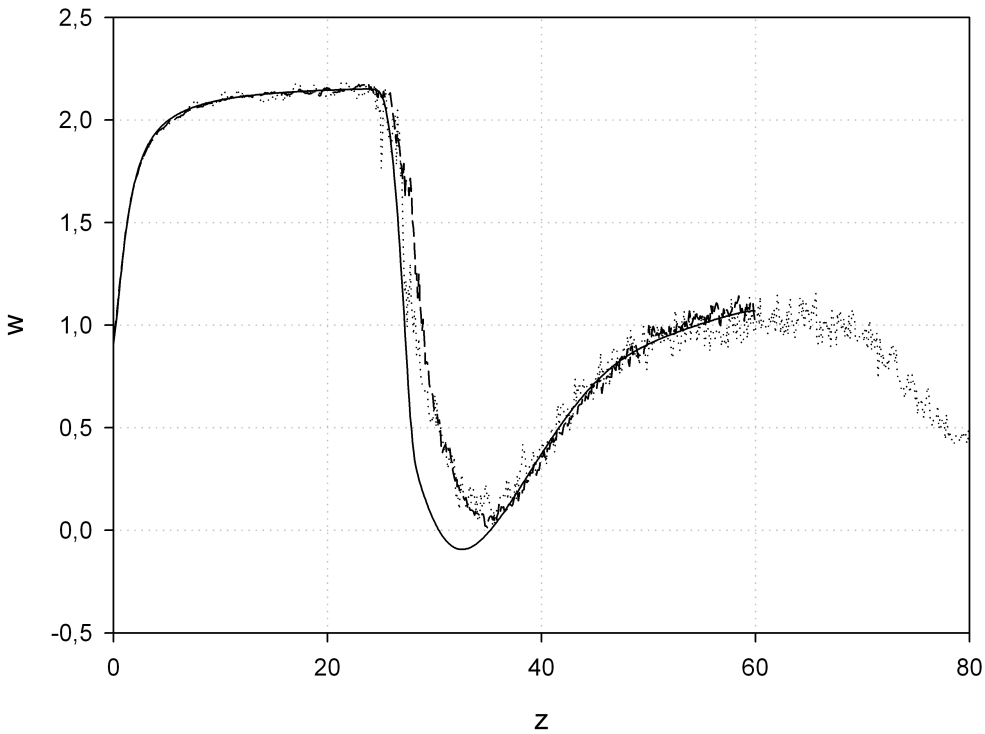

FIGURE 2. The distributions of axial velocity along the centerline (solid line – NS-algorithm, dotted line – DSMC-approach (BC1), dashed line –DSMC-approach (BC2).

Figure 2 illustrates the distributions of axial velocity normalized by c_0 along the centerline (here and below the coordinates *r*, *z* are normalized by the radius of nozzle exit r_e). The profiles obtained by NS-algorithm as well as by DSMC-approach with two variants of setting the boundary conditions on surface 4 are shown. As it was indicated above, both profiles obtained by DSMC approach are in good agreement with each other inside the region $z \leq 60$. The significant drop of velocity at $z > 60$ reveals the mentioned above disturbances of the flow field caused by the boundary conditions of type BC1 in the vicinity of surface 4. The boundary conditions of type BC2 seem to give quite reasonable results. In the jet core (z < 25) the NS- and DSMC-data are in good agreement with each other.

As it is seen from Fig. 2, in the entire region the axial velocity obtained by DSMC-approach is positive in contrast with the profile predicted by NS-algorithm with negative velocity for $30.4 < z < 35.2$.

The existence of region with negative axial velocity indicates the formation of the recirculation zone in the flow field that may be seen from Fig. 3 where the streamlines picture for NS- and DSMC (BC2)- distributions of parameters are shown. In the NS-picture (Fig. 3a) the recirculation zone is well expressed, – its radius is about a half of the radius of the Mash disc. The DSMC-streamline picture (Fig. 3b) is in good agreement with those obtained by NS-algorithm in almost all the domain except the region behind the Mach disc where no recirculation zone is formed.

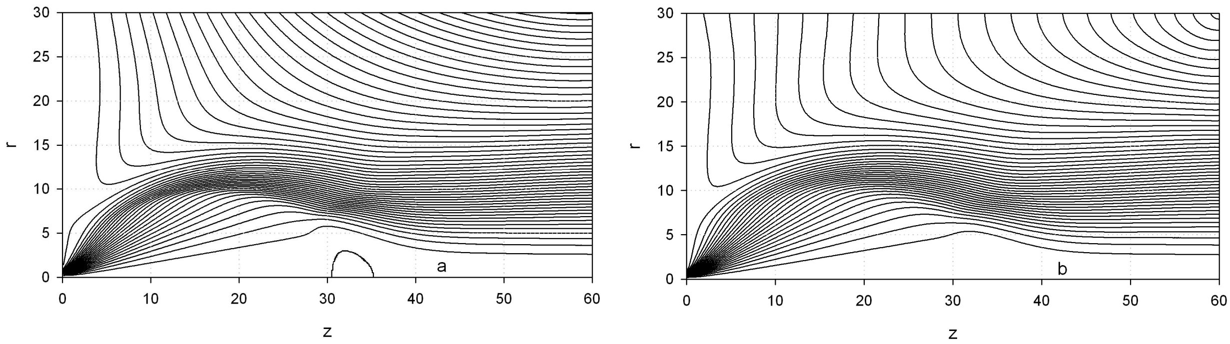

FIGURE 3. The streamline pictures in an underexpanded jet behind the sonic nozzle predicted by NS- (a) and DSMC- (b) algorithms.

Figure 4 illustrates the field of isochores obtained by NS – (Fig. 4a) and DSMC- (Fig. 4b) algorithms. The pictures reveal the structure typical of underexpanded free jet with formation of Mach disk, barrel shock and triple point. Both pictures are in agreement with each other though in DSMC-field the track of boundary conditions type BC2 is visible in the near vicinity of surface 4 (see Fig. 1) as well as in the right upper corner of the domain. Of course, these small disturbances have negligible effect on the main jet structure. The DSMC-approach predicts noticeable larger widths of the Mach disk and of the shock layer between the barrel shock and the jet boundary compared to those obtained by NS-algorithm.

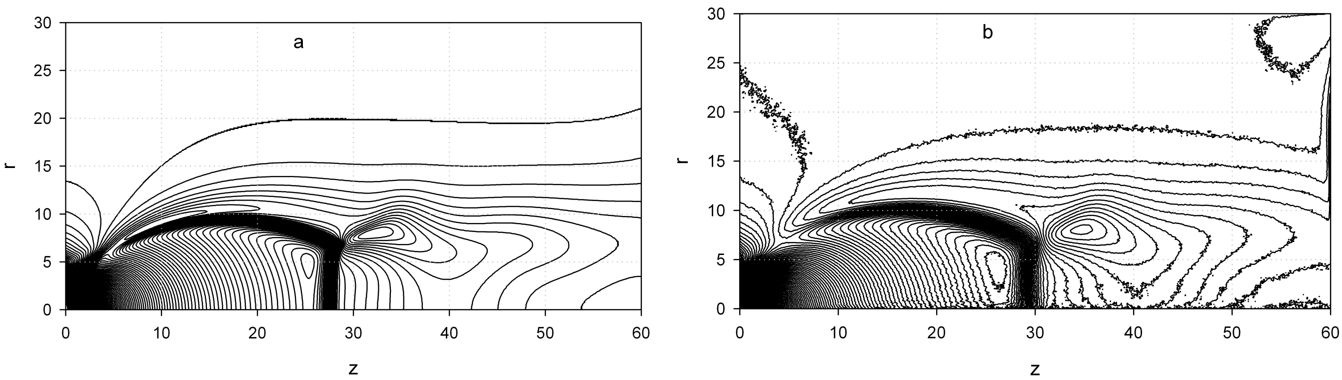

FIGURE 4. The field of isochores in an underexpanded jet behind the sonic nozzle predicted by NS- (a) and DSMC- (b) algorithms.

To understand the reasons of fundamental difference of the flow fields obtained by NS- and DSMC-approaches (see Fig. 3) let's analyze the applicability of Navier-Stokes equations for the description of the studied flow. It is well known that the Navier–Stokes equations cannot adequately describe the gas flow in a region several mean free paths thick near the surface (the Knudsen layer) as well as in front of a strong shock wave [8]. The latter fluid-dynamic element is present in the flow of our concern. We searched for the regions of the flow where the applicability of the continuum approach is questionable by applying the procedure proposed in [11], based on the gradient-length local (GLL) Knudsen number:

$$(Kn)_{GLL} = \frac{l}{Q}\left|\frac{dQ}{ds}\right| \tag{1}$$

where l is the local mean free path, Q is the flow property (density or temperature), and s is some distance between two points in the flow field. The applicability condition is [11]

$$(Kn)_{GLL-D} < 0.05 \tag{2}$$

where $(Kn)_{GLL-D}$ is the Knudsen number based on the density ($Q \equiv \rho$). The estimation of the Knudsen number $(Kn)_{GLL-D}$ was made by the relation

$$(Kn)_{GLL-D} = \frac{l}{\rho}\sqrt{\left(\frac{\Delta\rho}{\Delta r}\right)^2 + \left(\frac{\Delta\rho}{\Delta z}\right)^2} \tag{3}$$

for all the points of the flow field.

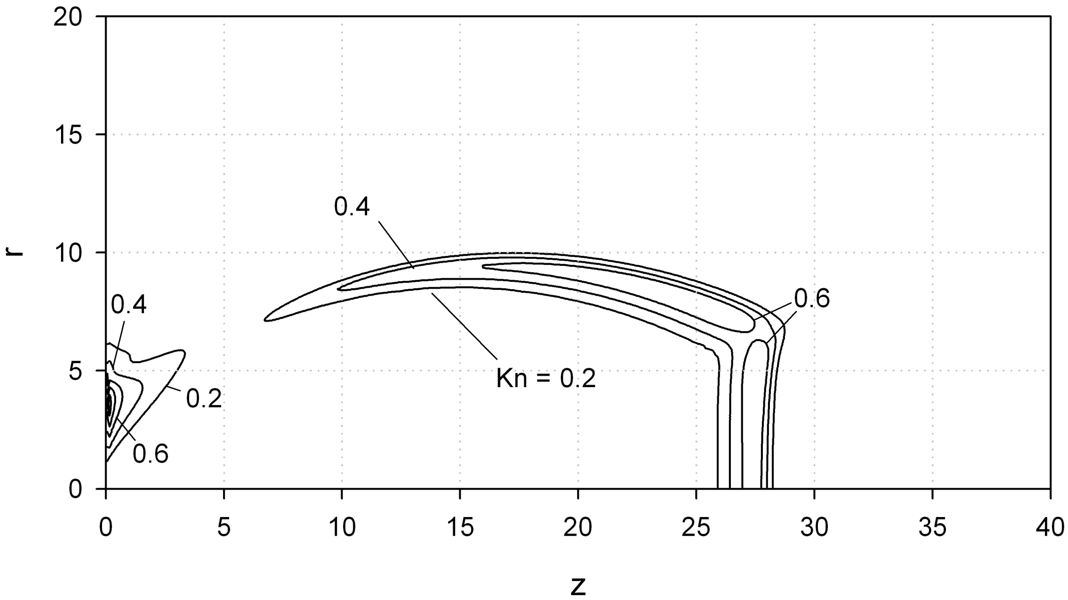

FIGURE 5. The distributions of gradient-length local Knudsen number $(Kn)_{GLL-D}$) in the flow field obtained by NS-algorithm.

Fig. 5 illustrates the distribution of gradient-length local Knudsen number ($(Kn)_{GLL-D}$) in the flow field obtained by NS-algorithm. The lines corresponding to the values of the Knudsen number 0.2, 0.4 and 0.6 are shown. All these lines correspond to the conditions in the flow field where the applicability of Navier-Stokes equation to the description of the flow is questionable (see condition (2)). The shown lines correlate well with the field of isochores plotted in Fig. 4a. In the region far from the nozzle lip the Knudsen number achieves its maximum value ($(Kn)_{GLL-D}$ = 0.77) at the point with coordinates z = 25.65, r = 7.65, i. e. in the vicinity of the triple point (see Fig. 4a). In the vicinity of the nozzle lip the maximum value of the Knudsen number is even higher ($(Kn)_{GLL-D}$ = 1.2). This value of the Knudsen number is realized at the point with coordinates z = 0.15, r = 3.55.

CONCLUSION

The main results of the performed study may be summarized as follows:

1. The flow in underexpanded jet of molecular gas behind the axisymmetric sonic nozzle was simulated by two approaches – in the frames of full set of unsteady Navier-Stokes equations and by the standard DSMC-approach.
2. The way of using the Navier-Stokes's profiles of parameters on permeable boundaries as the boundary conditions for DSMC-approach was tested and found to be appropriate.
3. For studied regime of underexpanded jet the Navier-Stokes equations predict the formation of recirculation zone behind the Mach disc. Similar features of the flow field were reported earlier in literature.

4. DSMC-approach to the flow with the same set of determining parameters predicts no formation of recirculation zone in the flow field.
5. The analysis of the applicability of the Navier-Stokes equations to the description of the considered flow made on the basis of gradient-length local Knudsen number reveals the regions where this applicability is questionable: the vicinity of the nozzle lip, the vicinity of the Mach disc including the triple point, and the shock layer between the barrel shock and the jet boundary.
6. The nature of the discussed effect, therefore, seems to be purely computational and caused by limited applicability of the Navier-Stokes equations for the description of the flow in the vicinity of the Mach disc with high gradients of the flow parameters here.

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